A microsimulation-based analysis of the price-of-anarchy on a Braess-like network

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SHORT SUMMARY
In the scientific literature the ratio between the total travel cost under a user equilibrium assignment and the total travel cost under a system optimum assignment is typically referred to as the Price of Anarchy (PoA). Recently, this concept has been attracting renovated attention due to the new opportunities offered by vehicles' connectivity and automation. The new technologies could allow individual routing suggestions centrally managed to achieve benefits in network performance. However, considering the infrastructure that such a system would need and the ethical implications it could have (related to privacy, equity, etc.), it is necessary to carefully quantify its actual benefits. Existing PoA related studies do not fully capture this essential realistic traffic dynamics. In this light, the present paper investigates the PoA over a Braess's-like network modified to include realistic road characteristics. The network is modeled and its traffic dynamics analyzed using a microscopic traffic simulation over a wide range of combinations of its input parameters. Results show that the PoA can be much higher than that obtained in theoretical studies. In addition, results are used to reveal some PoA features of real networks and propose further research directions.

Keywords: Price of Anarchy, Connected and automated vehicles, Vehicle routing, User equilibrium, System optimal assignment.

1. INTRODUCTION
Technologies for vehicles connectivity and automation are expected to reshape transportation as it is known today. A typical example is the possibility to provide individual routing guidance or network access permission to connected and automated vehicles (CAV) with the objective to maximize road network efficiency. In this sense, in a few years from now, new technologies may allow for the reevaluation of the existing state of the art for network-level traffic management in order to move from inefficient selfish user behavior to more coordinated and efficient control. This concept is all but new. In 1952 J.G. Wardrop was already suggesting that a central authority could distribute vehicles over the road network in an optimal way (defined "system optimum" (SO) or "social Wardrop equilibrium") in order to increase overall network capacity (Wardrop, 1952a, 1952b). Wardrop also introduced the idea to quantify the loss in transport efficiency due to the lack of coordination that later has been named as "Price of Anarchy" (PoA) which is defined as the ratio of the total travel cost under a user equilibrium (UE) assignment over the total travel cost under a SO assignment.
After Wardrop's seminal work, several authors have tried to elaborate on this concept and on the possible role of prescriptive routing to reduce travel costs (Roughgarden, 2005). However, the magnitude of possible improvements (and thus of the PoA) is not well understood yet. In the literature, there are two main branches of the PoA-related studies. One branch is from computer science domain and has mainly concentrated on theoretical aspects of the PoA features based on

analytical approaches (Anshelevich and Ukkusuri, 2009; Colini-Baldeschi et al., 2017; Correa et al., 2008, 2004; Koch and Skutella, 2009; Roughgarden, 2005; Roughgarden and Tardos, 2002). The other branch has collected empirical evidence for real transportation networks (Grange et al., 2017; Levinson, 2010; Monnot et al., 2017; Thunig and Nagel, 2016; Youn et al., 2008; Zhang et al., 2018). Analytical research works typically use linear or polynomial static cost functions (such as the Bureau of Public Road – BPR – cost function) to estimate the link travel time, which has proven to be not sufficiently accurate to describe realistic traffic behavior. In fact, in order to properly estimate the PoA one should take into account relevant traffic dynamics such as flow breakdown, link interdependence, demand elasticity, spillbacks, and gridlock effects, which cannot be reproduced by using a static cost functions based approach. Existing studies that include some traffic dynamics in the PoA analysis show controversial results. It is still not yet clear what reasons govern the PoA values, especially under detailed consideration of driving behavior. With new technologies possibly enabling in the near future what Wardrop had postulated almost 70 years ago, further investigations on the PoA and the mechanisms governing it under realistic traffic conditions seem necessary.

The present study is making a first step in a more thorough investigation about the PoA using a microscopic simulation model, which offers a good compromise between flexibility and realism.

Because of the highly detailed modeling approach and difficulties to calculate SO using microsimulation model we start with a simple network topology – a well-known 5-link network from the Braess paradox example. Braess paradox is the counterintuitive situation when an addition of a new link to the network decreases its efficiency in the UE state (Braess, 1968). This case is a typical example of PoA appearance since the situation can be improved by rerouting traffic to avoid the usage of that link. The network and the simulation tests were designed and implemented using the Aimsun commercial simulation software (Aimsun Next).

To reveal the relation of the PoA and driving behavior (i.e. the parameters of the models regulating vehicles' behavior (car-following, lane-changing and other models) the study has been repeated over 16 combinations of the most important car-following and lane-changing parameters with quasi-random values within a realistic range. The study is conducted for a fixed level of traffic demand, as the analysis of the relation between PoA and the traffic demand is out of the scope for this paper.

The contributions of this paper are twofold. First, we expose some of the PoA features related to realistic traffic dynamics and quantify their magnitude. Second, we show the variability in the PoA as a function of different aspects related to both network topology and drivers' behavior.

2. NETWORK AND METHODOLOGY

Since the publication of the original paper in 1968, the simple 5-links Braess network has become the benchmark for PoA analysis. Several studies show that the topology of Braess's network has fundamental significance for PoA existence (Zverovich and Avineri, 2012; Milchtaich, 2006).

In the present study it was necessary to modify the original network to enable the simulation of relevant real-word traffic phenomena. In particular the network, presented in Figure 1, includes a bidirectional 5th link leading to four possible routes instead of the three in the original case, two roundabouts and a traffic light. To create more variability between links, link 3 has the traffic light with a cycle of 60 seconds and green period of 30 seconds. The simulation time is 1 hour with the first 20 minutes considered to be the loading phase (i.e. warm-up period) and not used in the analyses. For the last 20 minutes there is no demand in order to capture the network unloading process. Hence, the period of interest, during which the data are collected, is from the 20th minute to the 40th minute.

Figure 1 Braess-like test network used in the study

2.1. Quasi system optimum traffic assignment.

Given the intrinsic difficulty to identify the SO using a traffic simulation model, an empirical approximation is determined adopting a Monte Carlo based approach. In particular, 512 simulations with different route proportions were ran. The proportions were selected in a quasi-random way and kept constant for the whole simulation period. The proportion approximating the SO configuration is the one with the minimum overall travel costs and is indicated here as qSO (quasi System Optimum). It is clear that the better the coverage of the proportion space, the closer qSO is to SO. In any case since the objective of the study is to understand the PoA magnitude, this approach is acceptable as it will provide a conservative estimation of the PoA
To deal with inherent stochasticity of simulation models, 5 replications with different random seeds of each of the 512 combination of flow proportions were carried out. The total travel time assigned to each flow proportion and used to select the qSO is the average travel time from the 5 replications.

2.2. User equilibrium assignment

To estimate the PoA value, the average travel time of the quasi-SO was compared to the average travel time related to UE. As an approximation of the UE, the DUE approach provided by Aimsun was used. The results of DUE in a network with the flow interactions described above are expected to be unstable both in the sense of convergence and across different random seeds. To estimate the PoA, we used then a distribution instead of average values. The number of iterations for the DUE case was 100, which was repeated for 25 random seeds.

3. RESULTS AND DISCUSSION

The main results achieved can be summarized as follows. First and foremost, the PoA always exists for the given network, regardless of the model parameters. In addition, the qSO is much more stable across parameter combinations compared to the DUE. This is an interesting result, showing that for the simple network used, the total travel time is less sensitive to the drivers' characteristics with the qSO assignment, while it is very sensitive to them in the DUE case. This sensitivity leads to high variations in the resulting PoA. Nevertheless, the average PoA consistently takes values in the upper boundaries of the estimations found in the literature, ranging from 1.6 to 2.6, but showing individual values as high as 3.4.

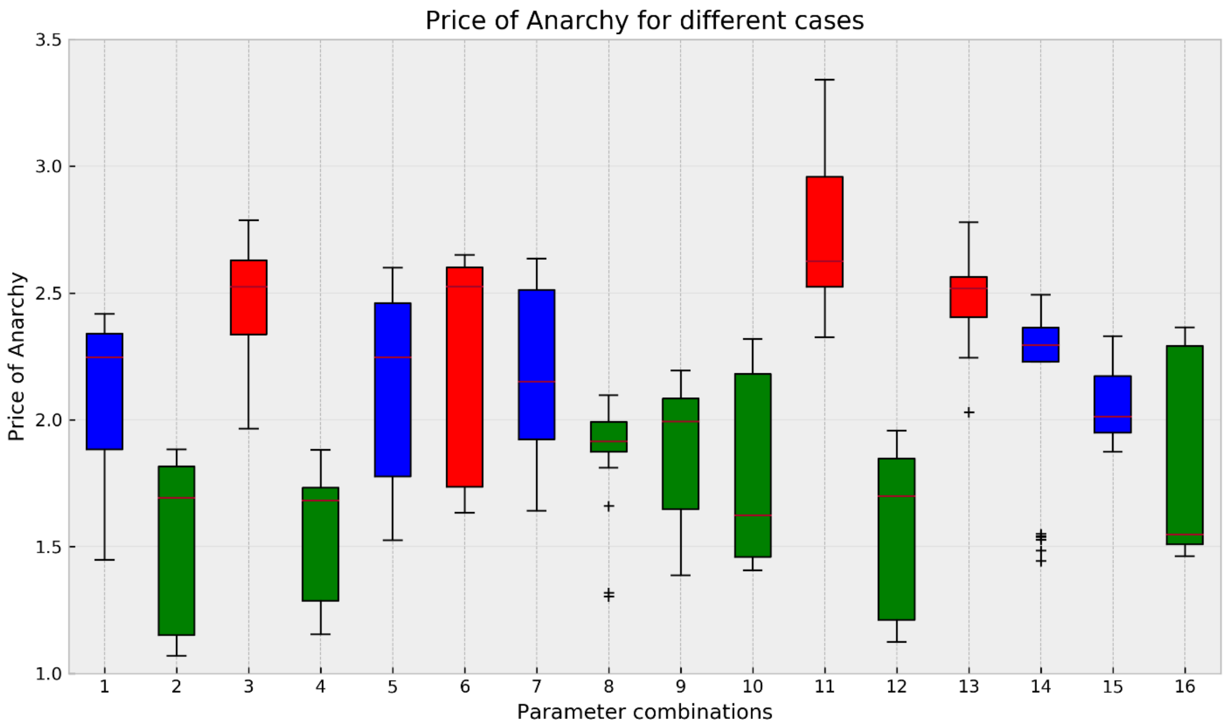

Figure 3: Values of the Price of Anarchy for the 16 parameter combinations.
The cases with median PoA less that 2 are colored green, the cases with the PoA more that 2.5 are colored red and the rest are blue

4. CONCLUSIONS

The present study focuses on the possible benefits from a coordinated routing system. The analysis is carried out on the simple but realistic network simulated with a state-of-the-art traffic simulation model. Results show that, for the specific network analyzed, the PoA consistently takes values in the upper range of the estimations found in the literature, varying in average from 1.6 to 2.6. Despite the fact that these results cannot be fully generalized, any network can have one or several locations similar to the studied one, where flow interactions decrease the potential network efficiency. Thus, it is of high importance to pay attention to the routing problem in future traffic management schemes.
From the analysis carried out, core reasons for the PoA are the marginal costs, flow interactions, and blockages. However, more research is needed to make this list exhaustive and better quantify the magnitude of the effects.
The results of simulations with different model parameters highlight the need for more realism on the assumptions about drivers' behavior; as well as the need for a proper model calibration to estimate the PoA on any particular network.

Further research is needed to confirm the results presented here on a real and more complex network and with variable travel demand. In addition, research is needed to identify efficient traffic management approaches able to actually achieve SO (or qSO as indicated in the present work). Moreover, as in some cases the SO cannot be achieved only by routing, research is needed on the combination of the routing and access control. This at the moment is a totally open research problem.

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